Filter Design: A Comparative Study Between the Cascade of Two First Order Filters and One Second Order Filter

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Abstract — Filters play an important role in everyday life, whether being used to tune in a radio station, scramble/descramble coded messages, or allow the ear to hear within a specific bandwidth. Obviously, some of these filters are more complex than others. Yet many times an effective complex filter can be created by the combination of several simple filters. In order to demonstrate this principle, one filter idea is proposed: Design two band pass filters, one using a singular second order band pass filter, the other using two first order filters, one low pass, the other high pass. It is realized that this represents an extremely simple case of filter design; however it demonstrates a very basic concept that can be applied to much more complex systems. We established a goal band pass of 230 Hz, a frequently encountered value in sound harmonics as well as the spacing between adjacent sub carrier RF signals [1]. Using LF 353 op-amps, the results show excellent agreement between the two filters, with only gain considerations as the only significant variance.

I. INTRODUCTION

Life needs filters. Many of our most highly regulated biological functions are dependent on the feedback generated by external stimuli. However, often times the amount of external stimuli is too massive to process efficiently. This is the fundamental importance of the filter: To provide only what we absolutely need process and disregard the rest. This principle holds true for today's electronic processors, because although it seems computers operate at such a high rate of speed with what seems to be unlimited memory, it is very easy to bog down a computer if it is asked to perform calculations for which the scope is too large- calculating 100000 points per second of a signal that lasts an hour, for example. By applying certain filters, these computations are made much simpler- taking only every tenth sample or every one hundred sample, for example. The type of filter used is a critical element to the efficiency with which it operates. If, for instance, human beings could hear all frequencies of sound, there would never be a moment that our brain was not processing information. However, if we were only able to process the frequencies of human speech and sound, approximately 20 to 4000 Hz, we would miss out on many wonderful sounds such as birds singing and bells ringing. It is for this reason that the human body uses a band pass filter for sound, ranging from 20 to 20000 Hz. But exactly how is this accomplished? One idea is that there are two filters, one that filters out every thing above 20000 Hz and one that filters out everything below 20 Hz. Certainly, it is equally effective to have only one filter that processes both extremes and permits only those between the extremes. This is the basis of this research.

The frequency we chose to isolate is 230 Hz, a common frequency encountered in many applications from sound to adjacent carrier spacing. We designed two different systems to filter this frequency. The first is a filter composed of a first order low pass filter in cascade with a first order high pass filter. The bandwidth of the low pass filter is 100 Hz and the bandwidth of the high pass filter is 600 Hz. The two signals are convolved to form a peak at 230 Hz. The second filter is a second order band pass filter which also isolates the frequency 230 Hz.

II. METHODS

The overall methodology developed for this experiment is consistent with the conventional methodology used for testing circuits. This methodology includes, but is not limited to, PSpice simulation, MATLAB and mathematical analysis, and realization of the physical circuit to extract experimental data. The recording of the experimental data is facilitated by the use of SONY/TEKTONIC circuit analysis equipment. More specifically, we used the AFG 310 arbitrary function generator to provide the input sine waves of varying amplitudes and frequencies, the PS 501-2 to provide the ±15V rails for the op amps, and the TDS 210 two channel digital real time oscilloscope to simultaneously record the frequency response of the circuit as well as the input signal. The individual circuits were treated identically, producing very similar results.

A. Analysis and measurements of the two first order filters in cascade

The first step in developing this model was to determine a frequency around which to build our filter. We decided on 230 Hz for reasons mentioned in the introduction. It was necessary to realize the need for a low pass filter to include 230 Hz and a high pass filter to include 230 Hz. The critical issue was to determine precisely where to place the cutoff frequencies for each filter such that, in cascade, they would produce an effective band pass filter. That is to say, produce a filter that would only allow the frequency of 230 Hz through, while attenuating (filtering) all other frequencies. This was accomplished using the mathematical relationships derived in figure 1 [2]. Simplifications were made in setting each of the resistor values equal to one another. This was also done for the capacitors. Once we were able to determine the values of the resistors and capacitors (see figure 2), those values were used to run simulations in PSpice to validate our calculations. The circuit was constructed and the experimental measurements were in agreement with PSpice modeling. Discussions on gain will be addressed further in RESULTS.

$$R = R_1 = R_2 = R_3 = R_4 \quad \mathit{and} \quad C = C_1 = C_2$$

$$v_1 = -\frac{Z_P}{R} V_{in} \qquad \mathit{where} \quad Z_P = \frac{R \big/ j\omega C}{R + 1 \big/ j\omega C} = \frac{R}{j\omega RC + 1}$$

$$V_{out} = -\frac{R}{Z_S} v_1 \qquad \mathit{where} \quad Z_S = R + \frac{1}{j\omega C}$$

$$V_{out} = \frac{Z_P}{Z_S} V_{in}$$

$$H(\omega) = \frac{Z_P}{Z_S} = \frac{R/(j\omega RC + 1)}{(j\omega RC + 1)/j\omega C} = \frac{j\omega RC}{j^2\omega^2 R^2 C^2 + 2 j\omega RC + 1}$$

$$H(\omega) = -\frac{\omega_0^2 RCs}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2} \qquad \mathit{where} \quad \omega_0 = \frac{1}{RC} \quad \mathit{and} \quad Q = \frac{1}{2}$$

Figure 1. The transfer function of the cascaded low-pass and high-pass filters.

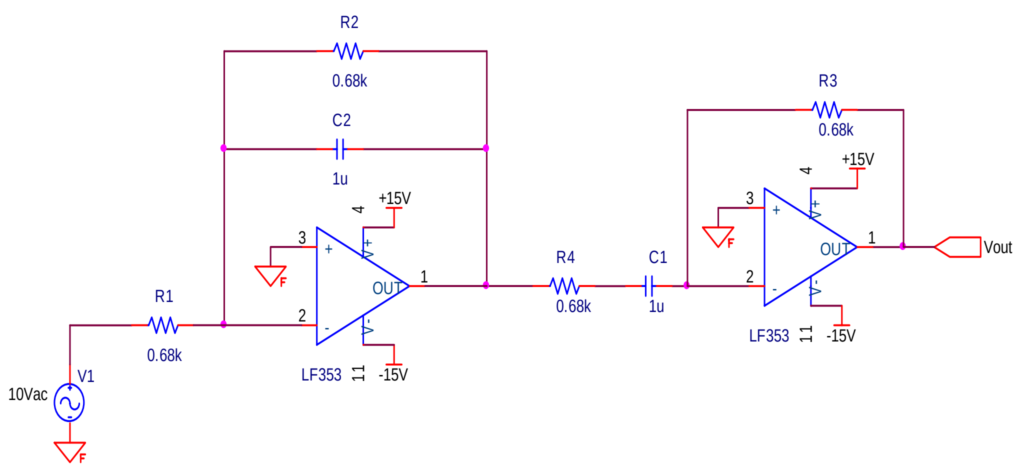


Figure 2. The realized circuit of the cascaded low-pass and high-pass filters.

B. Analysis and measurements of the 2nd order band pass filter

After deriving the transfer function necessary to generate the desired frequency response for the band pass filter created from the two first order filters, we used a similar approach to develop the RC relationships needed to generate the same band pass for the second order filter. Figure 3 shows the derivation of the transfer function as well as the conditions necessary to control the width of the band pass filter, i.e. the band width. The realized circuit can be seen in figure 4. Again, excellent agreement was achieved between experimental data and the simulated model.

II. METHODS (Cont.)

$$v_1 = i_2 Z_2$$
$$v_1 = v_2 + i_1 Z_1$$
$$v_1 = V_{in} - IR_1$$
$$v_2 = -i_2 R_2$$
$$I = i_1 + i_2$$
$$V_{out} = -\frac{R_2}{Z_2} v_1$$

$$v_1 = V_{in} - IR_1$$
$$v_1 = V_{in} - (i_1 + i_2)R_1$$
$$v_1 = V_{in} - \left(\frac{v_1 - v_2}{Z_1} + \frac{v_1}{Z_2}\right)R_1$$
$$v_1 = V_{in} - \left(\frac{v_1 + \frac{v_1 R_2}{Z_2}}{Z_1} + \frac{v_1}{Z_2}\right)R_1$$
$$v_1 = V_{in} - \left(\frac{v_1 Z_2 + v_1 R_2 + v_1 Z_1}{Z_1 Z_2}\right)R_1$$
$$v_1 + v_1 R_1\left(\frac{Z_2 + R_2 + Z_1}{Z_1 Z_2}\right) = V_{in}$$
$$v_1 = \frac{V_{in}}{1 + R_1\left(\frac{Z_1 + Z_2 + R_2}{Z_1 Z_2}\right)}$$
$$V_{out} = -\frac{R_2}{Z_2} v_1$$
$$V_{out} = -\frac{R_2}{Z_2}\left[\frac{V_{in}}{1 + R_1\left(\frac{Z_1 + Z_2 + R_2}{Z_1 Z_2}\right)}\right]$$
$$H(\omega) = -\frac{R_2}{\frac{1}{j\omega c_2} + \frac{R_1}{j\omega c_2}\left(\frac{\frac{1}{j\omega c_1} + \frac{1}{j\omega c_2} + R_2}{\frac{1}{j\omega c_1}\cdot\frac{1}{j\omega c_2}}\right)}$$
$$H(\omega) = -\frac{j\omega R_2 C_2}{1 + j\omega R_1 C_2 + j\omega R_1 C_1 + j^2\omega^2 C_1 C_2 R_1 R_2} = -\frac{\omega_0{}^2 R_2 C_2 s}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0{}^2}$$
$$\text{where} \quad \omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad \text{and} \quad Q = \frac{\omega_0 R_1 R_2 C_1 C_2}{R_1(C_1 + C_2)} = \sqrt{\frac{R_2}{R_1}}\left(\frac{\sqrt{C_1 C_2}}{C_1 + C_2}\right)$$

Figure 3. The transfer function of the 2nd order band-pass filter, showing the expression for ω_0, which is the desired frequency, and Q which controls the bandwidth

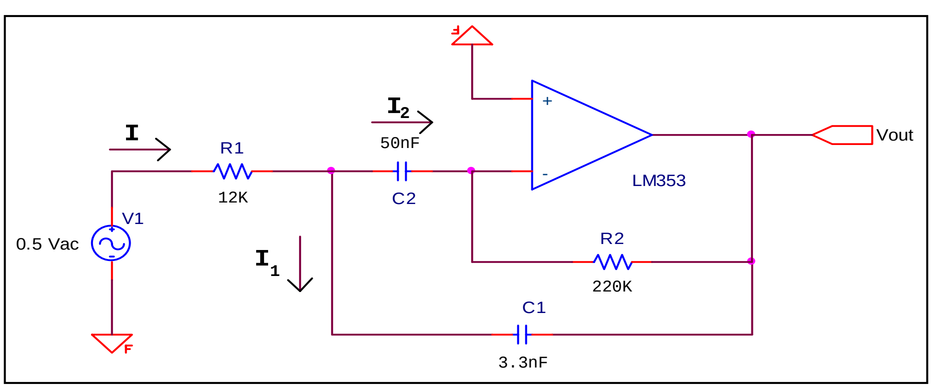
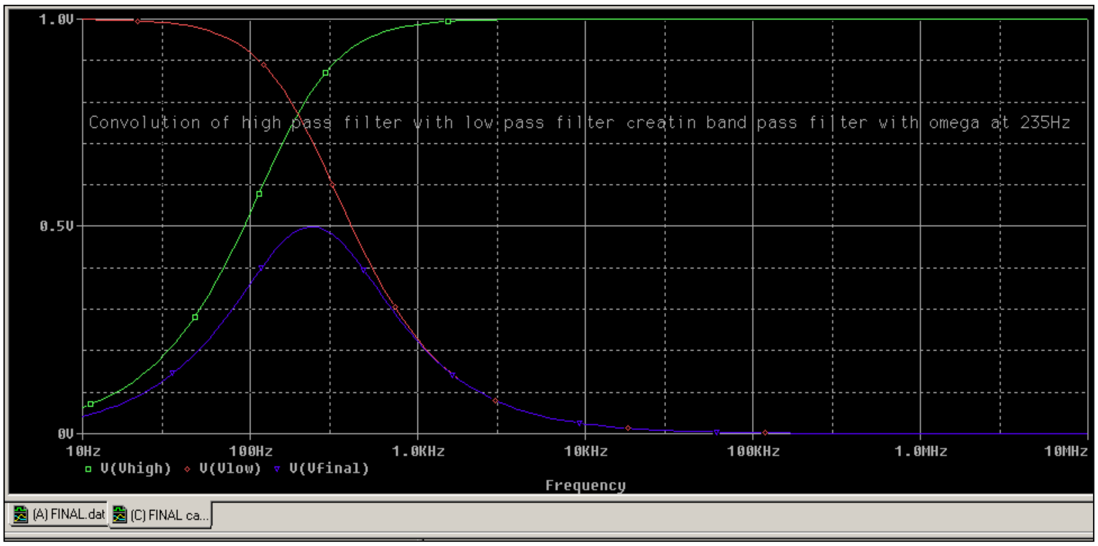


Figure 4. The realized circuit of the 2nd order band-pass filter.

III. RESULTS

A. Analysis and measurements of the two first order filters in cascade

As mentioned earlier excellent results were achieved performing this circuit analysis. Much of the data obtained through measurement was in very good agreement with the PSpice model. In figure 5, the PSpice simulation for the low pass/high pass (LPHP) filter, the expected frequency of 235 Hz is validated.



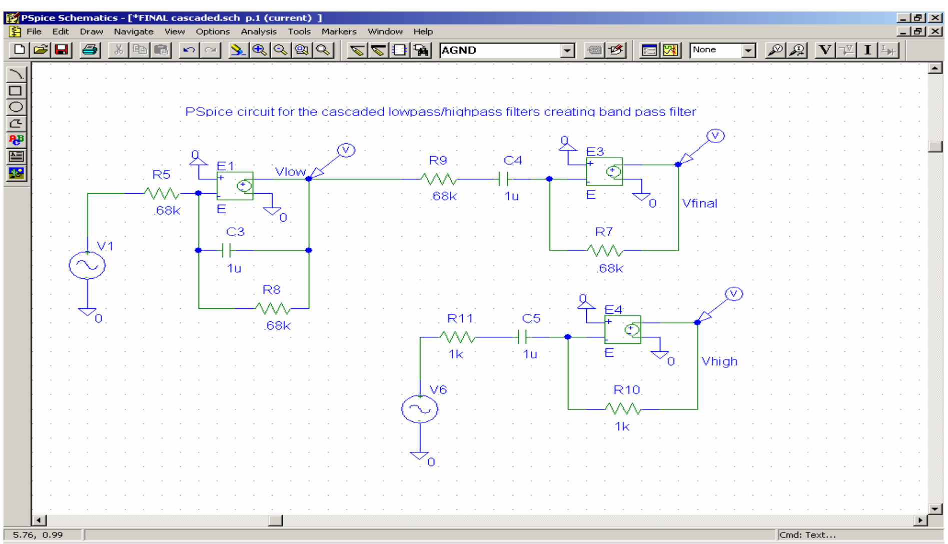


Figure 5. PSpice simulation of the LPHP filter.

These values are further established by carefully measured experimental data that further substantiates the validity of the results. Figure 6 is a graphical representation of the measurements made in conducting this experiment. The actual data points can be viewed in appendix A.

The LPHP yields a Frequency Response $H(j\omega) = \dfrac{j\omega RC}{(j\omega)^2(RC)^2 + 2(j\omega)(RC) + 1}$. To simplify the expressions for the LPHP, each of the R values were set equal to one another at 0.68kΩ. It is important to state at this stage that any gain in the value of Vout can be modified by changing the ratio of R7 to R5 in the PSpice model. (See figure 7.) This is important because we will see a difference between the output voltage from the 2nd order band pass filter and this LPHP filter. (See figure 8.)

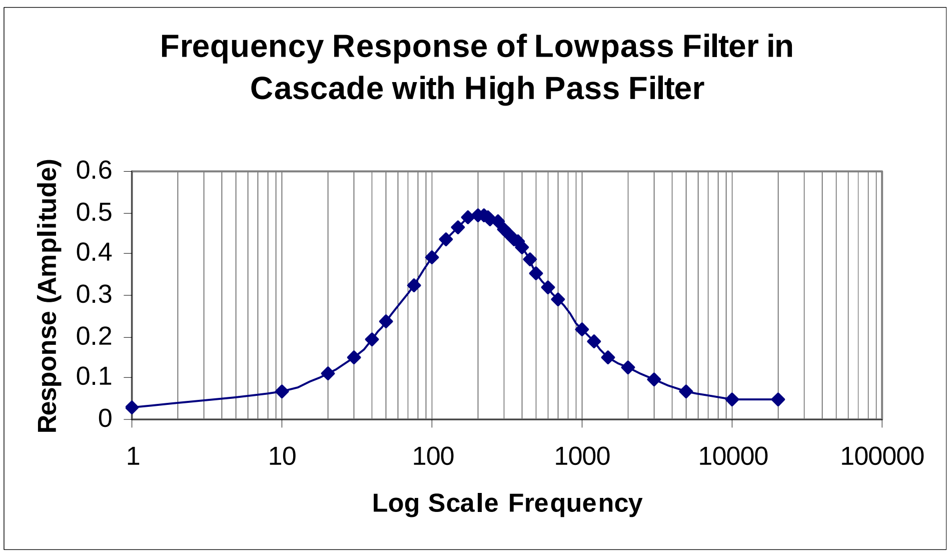


Figure 6. Graphical Representation of experimental data. Note the similarity of this curve to that of the PSpice model.

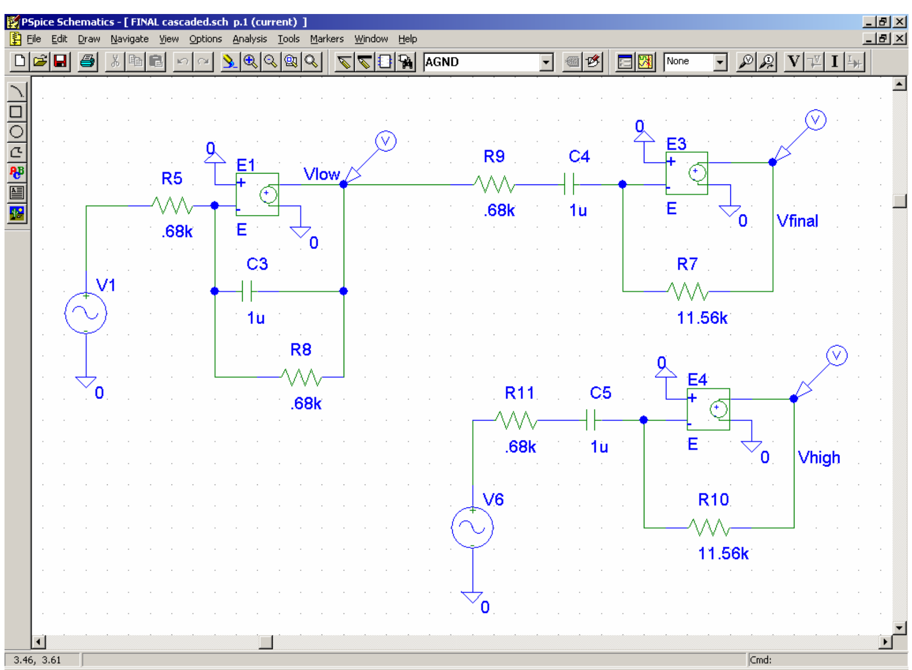


Figure 7. PSpice circuit with redefined R7 value to indicate a gain of 17 resulting in a Vout of 8.5V.

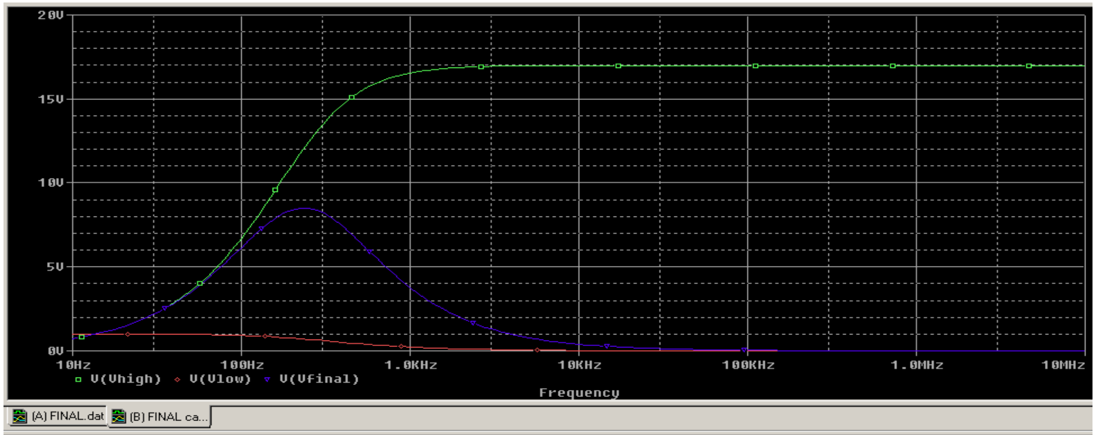


Figure 8. Frequency response showing a gain of 17. This is shown to attribute the difference in Vout values between the LPHP and the 2nd order band pass. Note the filter remains intact but the output value has increased to 8.5V. Also note the new output value from the high pass filter showing the gain of 17.

The most widely used graphical representation of the frequency response $H(\omega)$ is the Bode plot in which the quantities $20\log_{10}|\ H(\omega)|$ are plotted versus ω, where ω is measured in radians per second [3]. This magnitude of this quantity is referred to a decibel (dB). Figure 9 and 10 show the PSpice Bode plot of the LPHP filter.

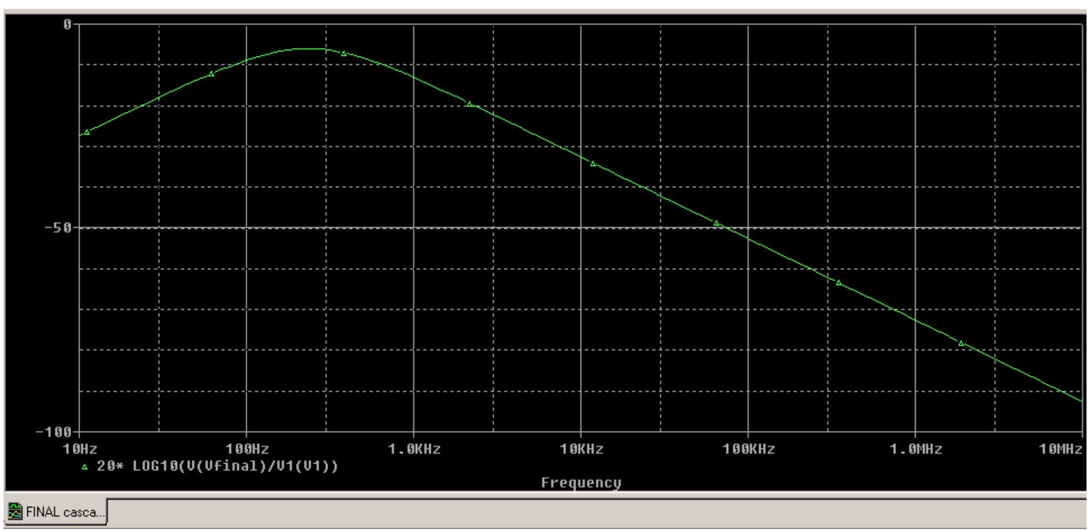


Figure 9. Bode plot of the LPHP according to PSpice.

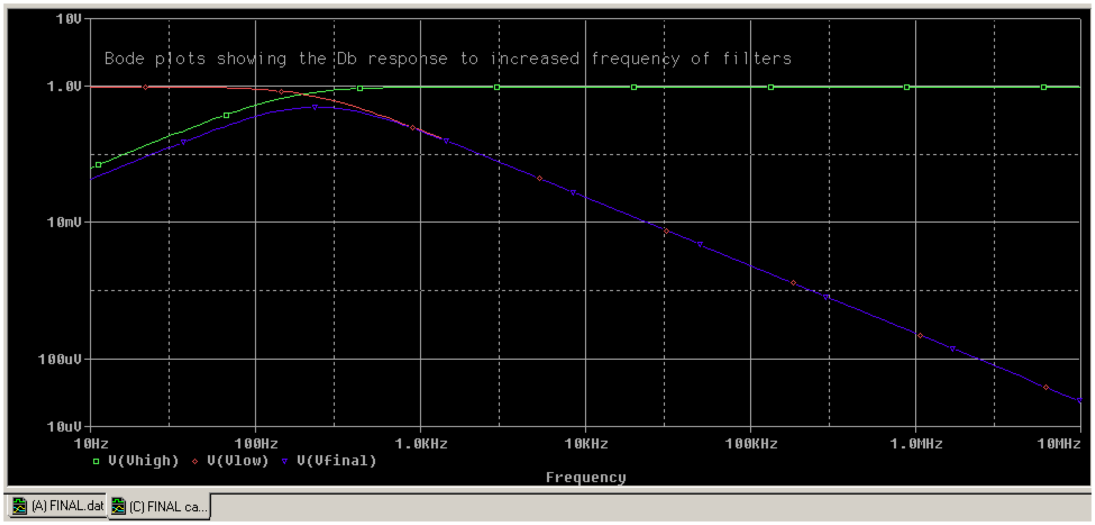


Figure 10. Bode plot of the individual filters in the LPHP filter according to PSpice.

We use MATLAB to further provide validation of our values. Using the frequency response equation for the LPHP filter $\frac{j\omega RC}{j^2\omega^2R^2C^2+2j\omega RC+1}$ we obtain the numerator coefficients of [6.8e-4 0] generated by R = 0.68k and C = 1uF. (The coefficients are the coefficients of the order of the (j ω) term.) We call this vector b. The denominator coefficients are similarly derived and are [4.624e-7 .00136 1], which we call vector a. In MATLAB, the command to generate the Bode plot is *bode(b,a)*, where b and a are given in vector form. Figure 11 is the Bode plot according to MATLAB. Bode plots provide information regarding both the attenuation of the signal as well as the phase. Note the difference in the scale; MATLAB uses (ω) in radians per second. As such, we need to divide through each of the (j ω) terms by 2 pi to get the frequency in Hz.

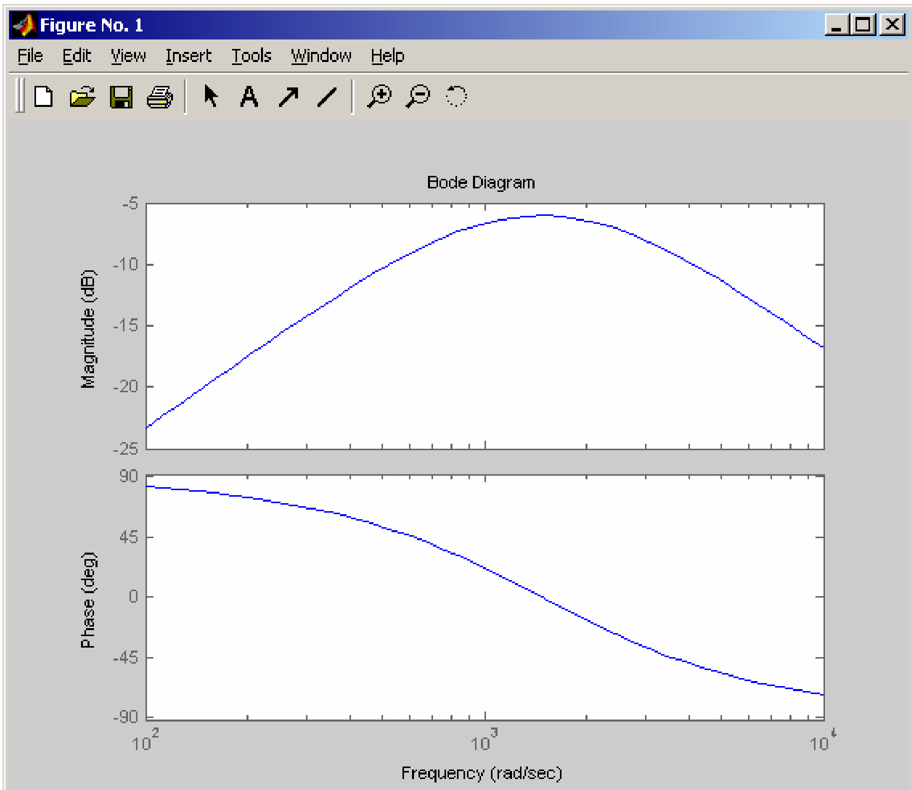
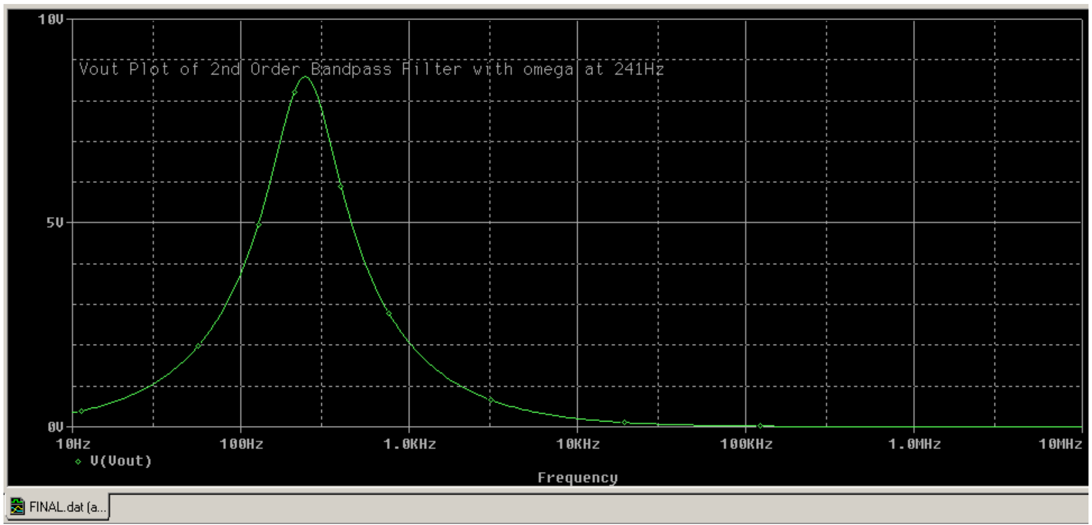


Figure 11. Bode plot of the LPHP according to MATLAB.

B. Analysis and measurements of the 2nd order band pass filter

The absolute bandwidth of an ideal band pass filter is given by $W_B = \omega_2 - \omega_1$. The band pass filter can also be defined by the 3dB bandwidth, using the magnitude spectrum of the signal. Although the transfer function for the 2nd order band pass filter is a bit more algebraically complex, it yields the same ω_0 [4]. However, the bandwidth is controlled by the Q value (Refer to figure 3.) Figure 12 shows the PSpice realized circuit along with the graphical representation of the frequency response of the circuit.

III. RESULTS (Cont.)



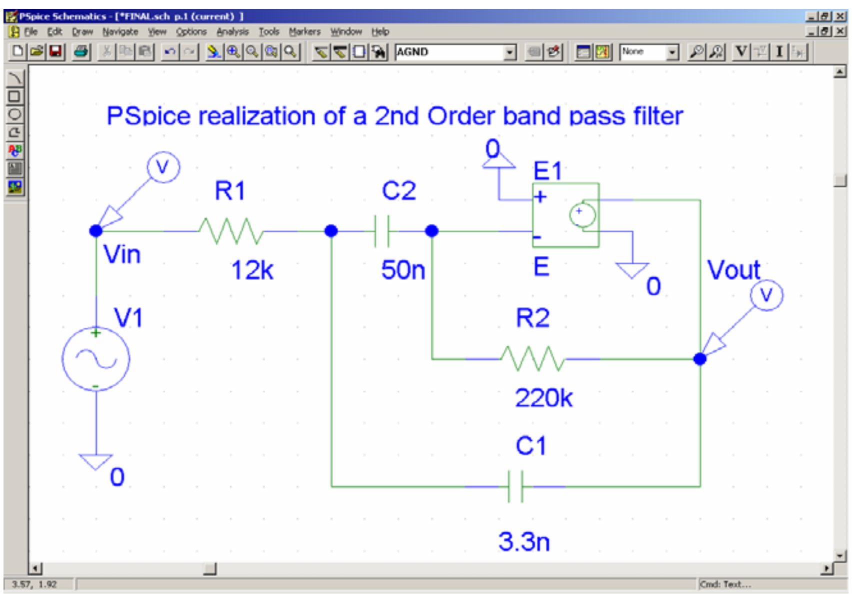


Figure 12. PSpice circuit with the corresponding Vout frequency response of the circuit.

The LPHP yields the transfer function

$$H(\omega) = -\frac{j\omega R_2 C_2}{1 + j\omega R_1 C_2 + j\omega R_1 C_1 + j^2\omega^2 C_1 C_2 R_1 R_2} = -\frac{\omega_0{}^2 R_2 C_2 s}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0{}^2},$$

$$\textit{where} \quad \omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \qquad \textit{and} \qquad Q = \frac{\omega_0 R_1 R_2 C_1 C_2}{R_1(C_1 + C_2)} = \sqrt{\frac{R_2}{R_1}}\left(\frac{\sqrt{C_1 C_2}}{C_1 + C_2}\right).$$

We compare the PSpice generated data to the experimental data and again see excellent agreement. Figure 13 shows the Vout frequency response of the realized circuit with an ω_0 = 235Hz as compared to the PSpice model with a prediction of 241Hz.

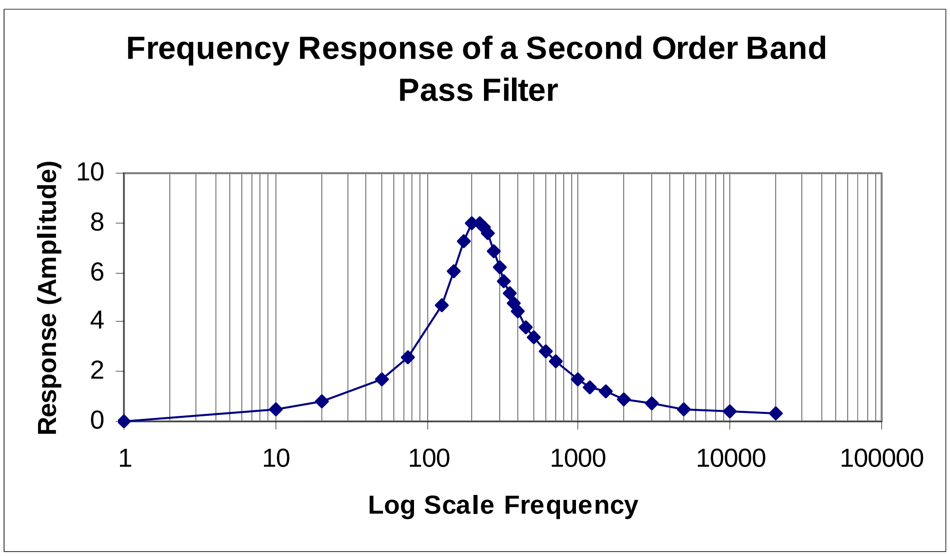


Figure 13. Graphical Representation of experimental data. Note the similarity of this curve to that of the PSpice model.

As with the cascaded LPHP filter, we look to the Bode plots to offer critical information regarding the frequency response of the 2nd order band pass filter. Figure 14 shows the PSpice generated Bode plot for this filter. Note the remarkable similarity to the Bode plot generated for the LPHP cascaded filter.

III. RESULTS (Cont.)

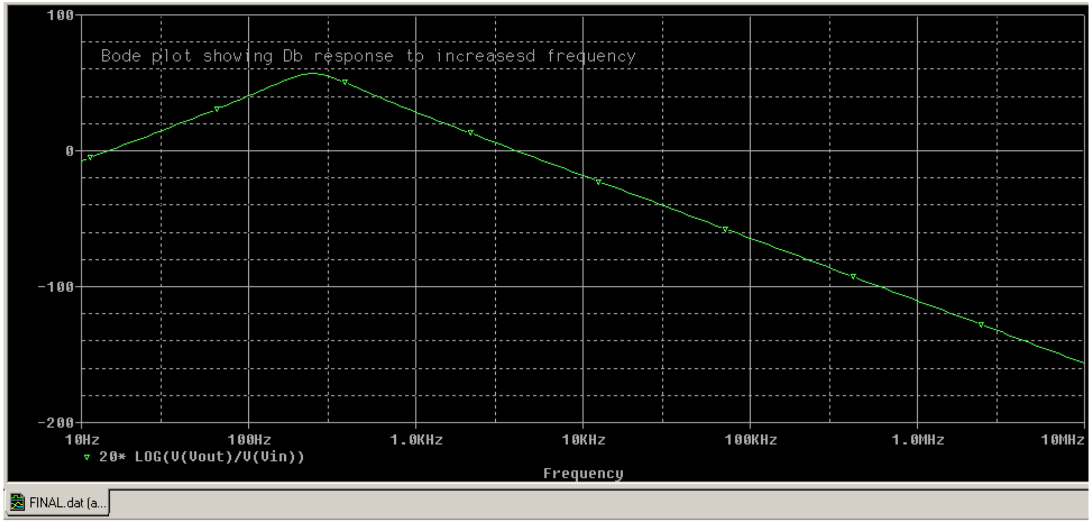


Figure 14. PSpice representation of the Bode plot for the 2nd order band pass filter.

We now look to MATLAB to provide further validation of the theoretical results. Figure 15 shows the MATLAB interpretation of the transfer function with numerator coefficients equal to b = [0.011 0] and denominator coefficients equal to a = [4.356e-7 6.396e-4 1]. As previously described, these coefficients are put into vector form and entered as *bode(b,a)*.

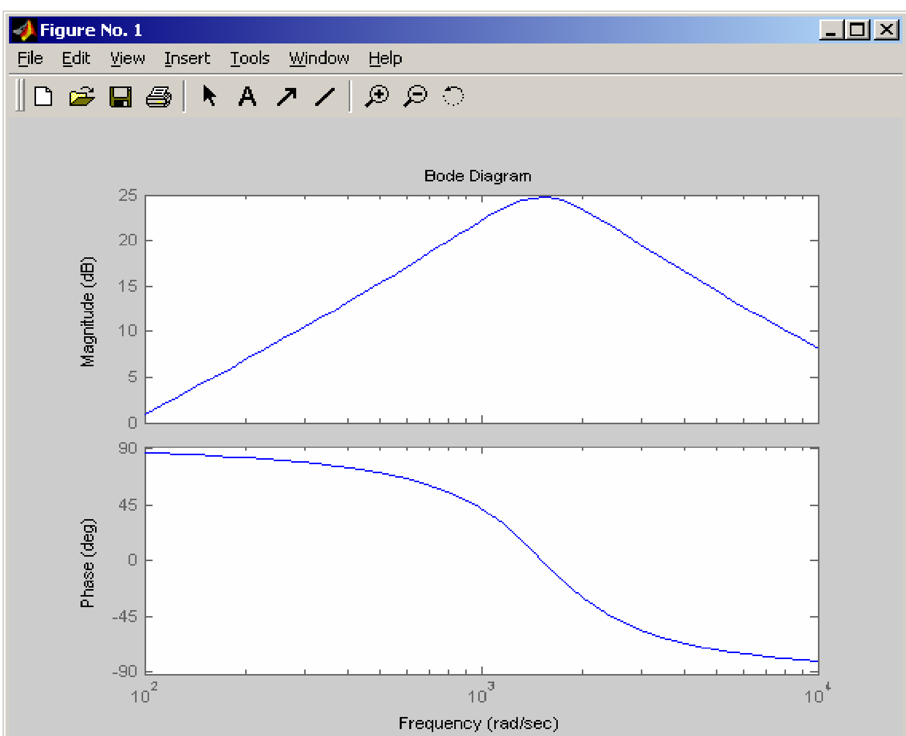


Figure 15. MATLAB generated Bode plot for 2nd order band pass filter. The scale is rad/sec.

Another analysis MATLAB performs is ZERO, POLE plots which help indicate the location of zeros and poles on the complex z plane. Figure 16 shows the zero pole plot for the transfer function of the second order band pass filter, with coefficients b = [0.011 0] and a = [4.356e-7 6.396e-4 1].

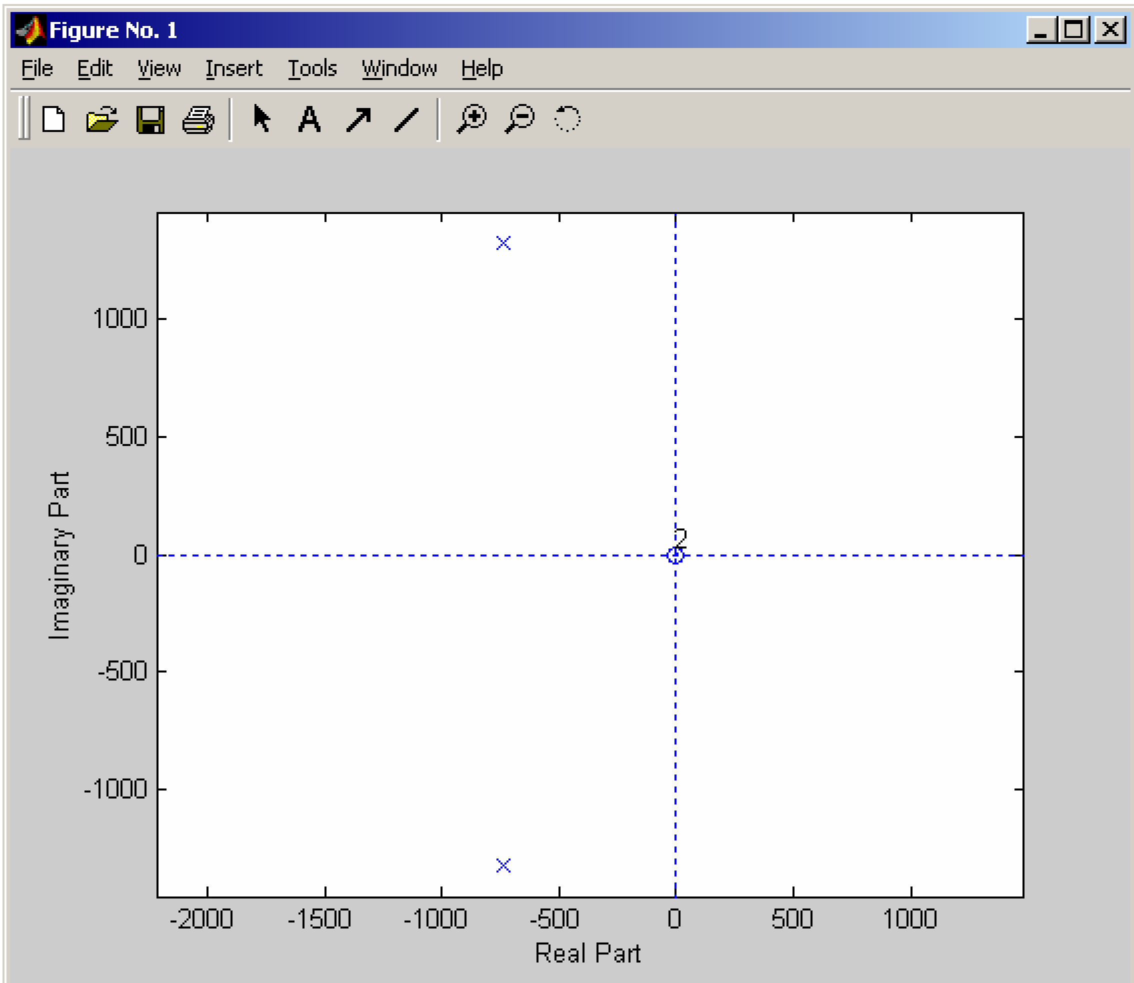


Figure 14. MATLAB rendition of the zero pole plot for the 2nd order band pass filter. Note that there are two zeros, both placed at the origin and two poles placed at -750 +/- j1350.

We note the location of the poles at $-750 \pm j1300$. The magnitude of this value is equal to $\sqrt{(750)^2 + (1300)^2} = 1500.833$ radians per second. Dividing this quantity by 2π yields the frequency 238.865 Hz, in excellent agreement with our measured values.

DISCUSSION

The band-pass filter is one of the most frequently used filters for common circuit functions. Their purpose is to isolate a certain range of frequencies where an output is present. The theoretical output for all frequencies outside the desired bandwidth is zero. Our goal was to use two different methods to isolate an output centered around 230 Hz. The results show that this goal was accomplished with minimal error.

One of the main purposes of this research was to demonstrate the simulation of high order circuits by cascading simple circuits together. The cascading of a high-pass and low-pass filter produced the exact same frequency response as the second order filter, as well as similar transfer functions. Although this only demonstrated two circuits in cascade, several others could easily be added to simulate 3^{rd}, 4^{th}, or 5^{th} order circuits.

In deriving the transfer function for the cascaded circuit, it became apparent that the gain of this circuit is dependent on the ratio of R_3/R_1. Once this was discovered, our attempt was to match the gain of the second order band pass filter. By inspecting the PSPICE simulated frequency response of the second order circuit, it was determined that the overall gain at ω_0 was 17. The ratio of R_3/R_1 on the cascaded circuit was then set to 17 and the frequency response was simulated, yielding a Vout of 8.5V. The reason for the gain only increasing half of what was expected was due to the difference in Q values. The Q value of the cascaded circuit is 0.5, while the second order circuit's Q value is twice that.

ACKNOWLEDGEMENT

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REFERENCES

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[3] Mark N. Horenstein. *Microelectronic Circuits and Devices, 2nd Edition.* Prentice-Hall, 1996. pp 808, 822-825.

[4] Hwei P. Hsu. *Signals and Systems*. Schaums Outline Series. McGraw-Hill, 1995. pp 230-233.

APPENDIX A

INPUT VOLTAGE = 0.5V
SECOND ORDER BANDPASS

Frequency	Amp	
1	0.04	p
10	0.5	
20	0.8	
50	1.7	p
75	2.55	
125	4.65	
150	6.05	p
175	7.25	
200	8	
225	8	
240	7.8	p
250	7.55	
275	6.85	
300	6.2	
325	5.65	p
350	5.2	
375	4.75	
400	4.4	
450	3.8	
500	3.4	
600	2.8	p
700	2.45	
1000	1.7	
1200	1.4	
1500	1.2	
2000	0.9	p
3000	0.7	
5000	0.5	
10000	0.4	
20000	0.3	p

LOWPASS/HIGHPASS

Frequency	AMP	
1	0.03	
10	0.07	
20	0.11	
30	0.15	
40	0.195	
50	0.235	p
75	0.325	
100	0.39	
125	0.435	
150	0.465	p
175	0.49	
200	0.495	
225	0.495	
235	0.49	p
245	0.485	
275	0.48	
300	0.46	
325	0.45	
350	0.435	
375	0.43	p
400	0.415	
450	0.385	
500	0.355	
600	0.32	
700	0.29	p
1000	0.22	
1200	0.19	
1500	0.15	
2000	0.125	p
3000	0.095	
5000	0.07	
10000	0.05	
20000	0.05	p

The 'p' designates a snapshot was taken from the oscilloscope for this frequency value. See the following results (15 pages).

APPENDIX B

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LF353

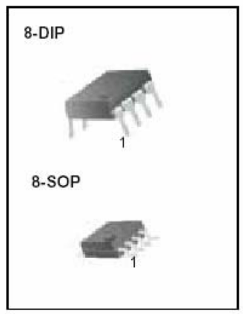
Dual Operational Amplifier (JFET)

Features

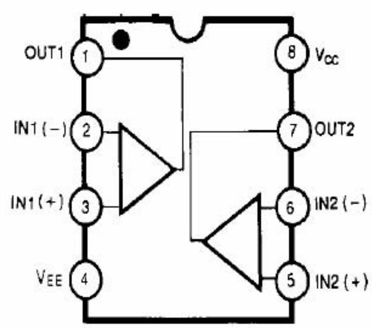
- Internally trimmed offset voltage: 10mV
- Low input bias current: 50pA
- Wide gain bandwidth: 4MHz
- High slew rate: 13V/μs
- High Input impedance: $10^{12}\Omega$

Description

The LF353 is a JFET input operational amplifier with an internally compensated input offset voltage. The JFET input device provides wide bandwidth, low input bias currents and offset currents.



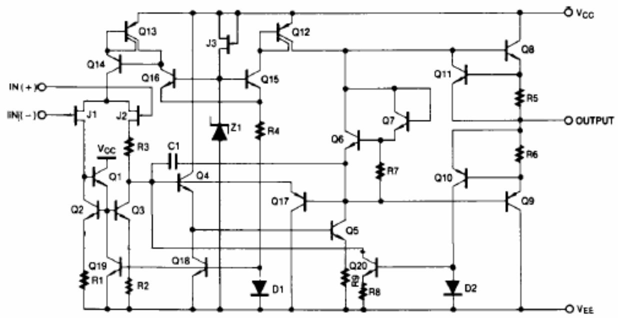
Internal Block Diagram



APPENDIX B

Schematic Diagram

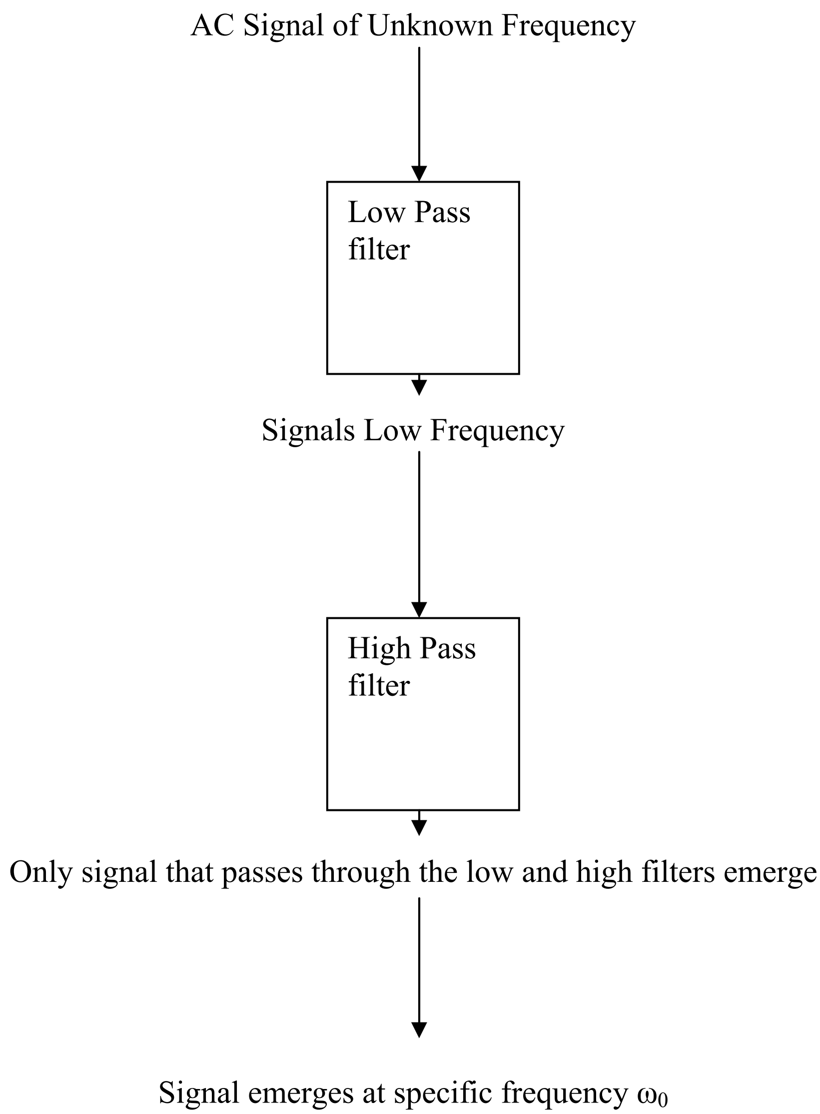
(One Section Only)



Absolute Maximum Ratings

Parameter	Symbol	Value	Unit
Power Supply Voltage	V_{CC}	±18	V
Differential Input Voltage	$V_{I(DIFF)}$	30	V
Input Voltage Range	V_I	±15	V
Output Short Circuit Duration	-	Continuous	-
Power Dissipation	P_D	500	mW
Operating Temperature Range	T_{OPR}	0 ~ +70	°C
Storage Temperature Range	T_{STG}	-65 ~ +150	°C

APPENDIX C
FLOW GRAPH
Cascaded First Order Filters



APPENDIX C (Cont.)

2nd Order Band Pass Filter